MACH'S "LECTURES ON PSYCHOPHYSICS"

By E. B. TITCHENER

In 1863 Ernst Mach, then *Privatdocent für medicinische Physik*, gave at Vienna a course of lectures on psychophysics.[1] We fortunately possess the reports of this course furnished by Mach to the *Oesterreichische Zeitschrift für Praktische Heilkunde* and there published in the same year. I believe that the lectures are practically unknown; and as they seem to me to be of more than merely historical interest I have made a brief summary of the contents of the reports,—a summary, be it remembered, of what is already part-summary and part-excerpt; the reader must expect nothing more than the barest outline. I have followed the divisions of the *Zeitschrift*, though I am not sure whether they represent separate lectures or available editorial space. Apart from the general headings, which I have ventured to supply, the rest of this paper is, then, a condensation of Mach's work.

The Science of Mind

§1. There is no special group of 'exact' sciences. Whether a science is or is not 'exact' depends solely on the stage of development to which it has attained. In fact, there is no reason why psychology (the science of the phenomena of the mental life) and psychophysics (the science of the interconnection of physiological and psychological phenomena) may not, if they follow the path they are already traveling, presently become in the fullest sense of the term 'exact' sciences.

[1]See my *Experimental Psychology*, II., ii., 1905, xlvi. The reports were reprinted in a little book, entitled *Vorträge über Psychophysik*, which I have never seen. For several years I sought it assiduously, but in vain; finally, I wrote to Mach himself about it. He replied that he knew of the existence of only one copy, which was in his own library; and with his usual kindliness—in such matters he was the soul of generosity—he offered, since I was seriously interested, to lend me the precious volume. I could not, of course, accept that risk; and as a reward of virtue I presently picked up the volume of the *Oester. Zts.* from which the present summary has been made (ix. Jahrgang, 146, 167, 202, 225, 242, 260, 277, 294, 316, 335, 352, 362). The volume, as it happens, contains also Mach's review of Helmholtz' *Tonempfindungen* (915, 930, 953).

What are today the 'exact' sciences? Mechanics, applied mechanics, physics in so far as it is applied mechanics, physiology in so far as it is applied physics, the science of man in so far as it is statistical,—all these disciplines rest upon exact laws. If no law can be found, that is either because there is no law to find, or because the law is too complex for immediate detection.

§2. Physical experiments made, in accordance with the Baconian rule of varying the magnitudes involved, with the view of establishing a natural law, always yield discrepant results. Extraneous influences are at work, influences which we sum up under the name of 'chance'. But chance itself is subject to law. Quetelet has shown that, as regards man, not only height, increase of population, etc., but also crimes, suicides, marriages, wrongly addressed envelopes, etc., are subject to numerical law; that is to say, the moral and intellectual elements of our social life, the psychological processes, are no less uniform than the rest.

We ourselves shall be mainly concerned with psychophysics, whose results (as Wundt has recently shown) are of practical importance for medicine. Meantime we must briefly consider Herbart's psychology, as the first attempt to deal with psychological phenomena from the mathematical point of view. The attempt is by no means ridiculous, since ideas, if not measurable magnitudes, at any rate vary in intensity and by that property admit of mathematical treatment.

Herbart's Psychology

§3. Herbart's psychology deals with artificially simplified phenomena, just as mechanics does. Ideas once aroused are, for him, indestructible; if driven from consciousness by other ideas, they persist as tendencies to ideation (cf. the law of inertia of matter). Disparate ideas are compatible; similar ideas are more or less antagonistic. Inhibition is mutual, and therefore minimal; its amount in the given case is inversely proportional to the intensity of the ideas concerned (Wundt has tried to show experimentally that the weaker of two ideas, even at an extreme difference of intensity, is never wholly suppressed); and the clearness of the ideational fragments remaining after it has done its work varies roughly as the square of the intensity; *i. e.*, an idea twice as strong as another remains about four times as clear as that other, and so attracts the attention.

A number of concurrent ideas may establish an equilibrium, with complete suppression of weaker ideas; mutual inhibition then operates as if these weaker ideas did not exist. Hence we find but relatively few ideas present in consciousness at any given time, and hence we are not burdened and restricted by the ideas we have forgotten. Equilibrium is not achieved in a

moment; the nearer it is approached, the slower is the advance towards it; indeed, it is never absolutely complete. In this way we account for the motility and variability of our internal states.

Ideas which concur in consciousness form permanent connections, and form them the more completely the freer the concurrent ideas are from inhibition. Here is the explanation of association and mediate reproduction. A series of ideas, whose earlier terms are increasingly inhibited as the series proceeds, allows only of progressively weaker connections between these earlier and the later terms; if the whole series has been inhibited, and presently a member returns to consciousness, the terms are reproduced in their original order. Herbart thus explains not only our remembrance of poetry but also the building up of our spatial perceptions.

Herbart's mathematical results come so near to the facts of experience that we must believe him to be on the right path. The good observer can, so to say, actually feel within him the struggle and mutual suppression of his ideas. But we turn now to experiments on sensation. If we find that sensations are measurable, we remove one of the principal objections to Herbart's psychology.

The Methods of Psychophysics

§4. The problem of psychophysics is to determine exact relations between stimulus and sensation by the way of observation and experiment. We have to measure both stimulus and sensation. The measurement of stimulus is simple; but sensation can be measured only by recourse to stimulus. It seems that we are involved in a circle.

Fechner, however, has shown us how to overcome this difficulty. We measure sensation by aid of the just noticeable difference of stimulus, which corresponds always with an identical increment of sensation, and thus furnishes us with a sensation-unit. We must, it is true, employ different sensation-units for the different classes of sensation, and cannot reduce them to such common terms as physics has found in mass, space and time; but there is ground for hope that a reduction may presently be effected.

The special methods of psychophysics seek to determine points on the path of a curve, *i. e.*, to ascertain for a selected number of stimulus-values those increments of stimulus which condition determinate increases of sensation. They are as follows.

Method of Just Noticeable Differences.—"For a series of stimulus-values x_1, x_2, x_3,.....we determine the just noticeable differences Δx_1, Δx_2,.....which make $x_1+\Delta x_1$ just distinguish-

able from x_1, $x_2+\Delta x_2$ just distinguishable from x_2, and so on, and which therefore in every case correspond with one and the same increment of sensation Δy. If it turns out, *e. g.*, that Δx_2 is twice as large as Δx_1, it necessarily follows that for the stimulus x_2 the rise $\Delta y/\Delta x$ is only half as large as for x_1. If we should find in general that the 2, 3, 4,.....*n*-fold stimulus has the 2, 3, 4,.....*n*-fold just noticeable difference, we could infer that $\Delta y/\Delta x = a/x$, *i. e.*, the increase would be inversely proportional to the stimulus-magnitude x. Mathematics would then draw the further inference that $y = a\ (\log x/b)$, where a and b are constant magnitudes."

Method of Average Errors.—"We take a constant stimulus x and try again and again to make a variable stimulus x_1 equal to it; we determine the error of every trial, and from a large number of these errors calculate the average error. In comparing our sensations we are thus subject to error for the reason that a series of accidental circumstances influences our judgment. For example, we take the two sensations y and $y+\Delta y$ and therefore also the two stimuli x and $x+\Delta x$ for equal, and so make the error Δy in estimating sensation and the error Δx in estimating stimulus. The error Δy, by the laws of probability, cannot exceed a certain magnitude. The average error will also, in a long series of observations, keep within certain limits of magnitude. It is, however, clear that one and the same error Δy will have corresponding with it a larger error Δx, the more slowly Δy varies with Δx, *i. e.*, the weaker the rise of the sensation-curve. Hence the average error in our estimation of the stimulus x is inversely proportional to the rise of the sensation-curve for the stimulus x."

Method of Right and Wrong Stimuli.—This method "is a modification (or rather, inversion) of the method of average errors."

The Facts of Psychophysics

§5. We pass from the methods to the facts of psychophysics.

I. *Weber's Law*, so named by Fechner, declares that the just noticeable difference is proportional to the magnitude of stimulus. Fechner in particular has shown that, within certain limits, the law holds in the most various departments of sensation.

(1) *Intensity of Sensations of Light.*—Here belong Fechner's experiments with cloudlike figures painted on white paper; Volkmann's experiments with shadows; Masson's observations of grey-ringed white discs. All experiments show that not absolute but relative differences are important for us. A stim-

ulus must increase by a determinate aliquot part if the increase is to be remarked; and the same stimulus-difference is noticeable or unnoticeable according to the magnitude of the stimulus.

The law does not hold for very high and very low degrees of illumination. Helmholtz' experiments with rotating discs seem to show that it holds, in any case, only approximately. Fechner contends, however, that the external stimulus must always be increased, for purposes of calculation, by the amount of the intrinsic retinal light (determined by himself and Volkmann as equal to the light intensity of a black velvet surface illuminated by a stearin candle at a distance of 9 ft.). The results of observation are in this way brought into much closer agreement with the law.

(2) *Intensity of Sound.*—The experiments of Fechner and Volkmann confirm the law.

(3) Our sensations of *tonal pitch* and *interval* follow the law (Herbart, Drobisch); our sensitivity to *color* does not.

(4) Weber found the law to hold for our sensations of *resting and lifted weights.* Fechner's numerous and exact experiments with lifted weights give a good agreement with the law if the weight of the lifting arm is taken into account.

(5) Fechner believes, as against Weber, that we may have an uniform sensation of *warmth* or *cold.* It seems that the law applies to temperatures which differ but little from the mean (unsensed) temperature; it certainly does not apply to those that depart widely from the mean.

§6. (6) According to preliminary experiments of Fechner's the law holds for *mixed* sensations (colors).

(7) It holds also (Fechner, Volkmann, Appel) for *distances* and *lengths,* which are probably the resultants of a complicated psychological process. In the case of distance, *e. g.,* the facts of observation are satisfied if we regard the average error as made up of a constant error, identical for all distances and due to the division of the retina into a finite number of sensitive elements, and a variable error, proportional to the distance.

(8) Czermak's work on the sensation of *time* is preliminary only. Mach's experiments with pendulums (1860) prove that the law applies to this sensation.

II. A second fundamental fact is the *Law of the Limen.* Psychophysics distinguishes stimulus and differential limens, intensive and extensive limens.

III. Fechner's *Parallel Law* (that reduction of sensitivity has, at least in many cases, the same effect as reduction of stimulus) needs further investigation. It seems to hold for lifted weights, but not for sensations of light.

Fechner's Interpretations

§7. Weber's Law may be formulated as $\Delta y/\Delta x = a/x$, where a is a constant. By purely mathematical reasoning we derive from this expression Fechner's metric formula $y = a \log (x/b)$, where b is a second constant. Since $x = b$ means $y = 0$, x must be $> b$ if y is to attain to a finite positive value; b then denotes the liminal value of x. The Law of the Limen would therefore be contained in Weber's Law if Weber's Law itself were unconditionally valid. As things are, the laws stand side by side, empirical and not contradictory.

The metric formula may be made definite if we take as unity the sensation which corresponds with some determinate stimulus-magnitude, and define a accordingly.

This constant a is a measure of absolute sensitivity; it is by no means to be confused with the liminal value b. At present, it is true, we cannot apply a measure of absolute sensitivity, since we cannot compare the magnitudes of the sensations which, in different states of sensitivity, correspond with the same stimuli.

Fechner's further arguments regarding the 'aggregate' sensation and its summation from single sensations are noteworthy and interesting; but neither his psychology nor his mathematics is free from objection.

§8. Fechner extends his metric formula, first, from differences of sensation to sensations of difference, which he thinks enter into the aggregate sensation, under certain conditions, as sensations of contrast; and secondly, in the instance of tones, from a single dimension—pitch or intensity—to the two dimensions taken together. Again, his arguments are open to objection. Mach (1862) applies the formula to our sensitivity for the change of position of straight lines; theory and experiment are here in good agreement.

Sight and Hearing

Our path now takes a new turn. Natural science begins always with problems that force themselves on the attention; involuntary enquiry precedes voluntary, as reflex precedes voluntary movement. Laplace's planetary theory arose from the insistent demand of the facts themselves. And, as in astronomy, so it is in psychophysics. Light-stimulus and sound-stimulus are both alike oscillatory in nature. Why, then, does tonal pitch obey Weber's Law, and color refuse to obey? Why does the tonal series show the periodicity of the octave, and the color-series lack periodicity? Why are tonal compounds analysable, and color-mixtures unanalysable?

These questions can, in large measure, be answered. We come presently to the special investigations (Young, Fechner,

Helmholtz). Here we must refer briefly to the laws of oscillatory movement under the influence of variable forces (Seebeck, Mach), laws which explain, among other things, the phenomenon of sympathetic vibration. Our interest lies in the theory of hearing; and we find in the tuned series of Corti's fibres an apparatus which, in terms of these laws, enables us to account for tonal analysis. The beats which arise when two near-lying tonal stimuli act on the same fibre, and which cause a disagreeable sensation, solve (or at any rate simplify) the problem of consonance and dissonance.

§9. Musical tones consist of fundamental and overtones, which latter determine timbre. If two or more musical tones are sounded together, we hear a dissonance or a consonance according as fundamentals, overtones or combinational tones do or do not engender beats. If we sound a tone with its first overtone only, and gradually raise the pitch of the tone to its octave, we experience a 'feeling of recurrence', of the periodicity of the octave-interval. Other intervals show analogous phenomena.

The theory of Corti's fibres was proposed, almost simultaneously, by Fechner and Helmholtz. How shall we test it? (*a*) Fechner suggested that, if single fibres are incapable of vibration, the ear will lack the sensations of the corresponding tones. Politzer confirms this suggestion. (*b*) Unilateral affection of fibres should bring it about that the two ears hear the same tone differently. Cases have been observed. (*c*) Any lessening of the capacity of the fibres for vibration should decrease our sensitivity to tonal differences. (*d*) Since the fibres are limited in number (about 3,000), our estimate of very small differences of pitch will perhaps prove to be subject to a special constant error (cf. what has been said of the estimation of spatial distances).

We have seen how the sensations of color differ from those of tone. We add (*a*) that the mixture of two or more spectral colors gives rise in all instances to a color which can be represented by white and an intermixture of a single spectral color; and (*b*) that any mixed color whatever can be produced by the mixture of three properly chosen spectral colors. Brewster thought, accordingly, that there are only three objective colors; but Helmholtz has proved the continuity of the spectral series. Helmholtz and Fechner (who, again, published almost simultaneously) therefore suggest, following Young, that there are three fundamental subjective colors, red, green and violet. But Helmholtz believes (with Young) that each one of these colors has its own nerve-fibre, while Fechner assumes three different kinds of process within the single nerve-fibre. The point at issue will be decided the one way or the other according as the retina of

the partially color-blind (who lack a green-sensation) turns out to be defective in anatomical or in chemical elements. Or we might examine the color-blind periphery of the normal retina.

Since every objectively simple color is subjectively compound, no one has ever seen a simple color. If we look at a light for some time through a red glass, and suddenly exchange the red for a green, we see a green of quite unusual purity.

In summary: "the eye complicates the objectively simple, the ear analyses the objectively complex. On the other hand there is also a real resemblance. We might say that the eye is a mosaic of light-ears, with the number of Corti's fibres reduced in every ear to three.—These differences seem to account for at least the greater part of the differences in the behavior of our sensations of light and sound."

§10. Helmholtz' extension of the doctrine of specific energies, which follows on his theories of color and tone, is to be regarded with caution. Helmholtz thinks that not only sensations but also qualities of sensation have their special nerves; since, however, we find in the nerves only electrical currents, these nerves must be like telegraph wires, and simply transmit certain signals that correspond with the sensations. We know, however, too little of nervous processes. What we know comes, it is true, by way of electrical experiments; but these experiments, delicate as they are, are also too crude; effects that seem to be physiologically the same need by no means be physically identical.

The Problem of Perception

We have already objected to Fechner's view of the aggregate sensation as a mere sum of simple sensations. Wundt has recently shown (1862) that between sensation and perception there lies in fact a series of psychological processes, which he calls 'unconscious inference.' Helmholtz had previously come to practically the same conclusion. Wundt, *e. g.*, has proved the dependence of the perception of distance upon accommodation and convergence; he has proved further that there are no 'identical' retinal points. All Wundt's experiments on visual space-perception indicate that the sensations of the two retinas come separately to consciousness, and only there are connected, by a sort of inference, to a stereoscopic perception; and what holds of stereoscopic vision holds also of mirroring and lustre.

What now is the function of these unconscious inferences? Kant regarded space and time as forms of perception given a priori: but why then should some sensations (sight, touch) fall into the spatial schema, and others not? Weber makes the brain repeat the peripheral arrangement of nerve-endings: but here is not even an explanation of tonal analysis, to say nothing of

space-perception; and even if the theory be taken to account for our separate hearing of partial tones, it would still be inadequate to space, since the two modes of perception are not identical. Herbart explains space-perception as a case of serial reproduction: but on this hypothesis, again, all the senses ought to be spatially competent.

§11. Lotze, in his doctrine of local signs (which may be either a system of concomitant sensations or a system of movements, or rather of impulses to movement), attempts a combination of the anatomical and the psychological views. Wundt's theory is an extension of Lotze's. The part which Wundt ascribes to muscular feeling (against Schiff) is rendered highly probable by the results of his experiments on the estimation of distance. His theory takes account not only of visual space-perception but also of the tactual space-perception of the blind and the seeing. It is supported by observations of pathological phenomena (paralysis of the ocular muscles). Wundt has further observed that the 'sensory circle' very quickly changes under the influence of therapeutic treatment; and narcotics affect the retina as well as the skin. It may be that a diminution of sensitivity means a diminution of the intensity of movement-impulses; in that event we need not, with Fechner, modify Weber's theory of the sensory circles.

Mind and Body

§12. The materialistic point of view is untenable; it simply puts new prejudice and untruth in the place of old. Fechner carried out all his psychophysical investigations without setting up any hypothesis of the nature of the psychical, and without making any assumption regarding the processes that lie between stimulus and sensation. He suggests, however, in his *Psychophysik*, that physical and psychical may be the same thing looked at from different points of view (cf. the concave and convex aspects of a circle). We may arrive at a like hypothesis by the road of physics. For the facts of physics indicate that we have to do with a finite number of entities and forces, with what are called 'atoms.' Try now as we may, we can frame no rational idea of an 'external aspect' of these atoms; we are forced back upon an 'internal aspect,' in some sort analogous to our own 'mind'. And indeed, how should we ever get to 'mind' at all if the germ of mind were not present in the atom?

Fechner also raises the question of the 'seat' of mind in the organism, and finds it in the whole nervous system. There are difficulties in this view, but they are not insuperable. Fechner further considers the lower animals and the plants, and refuses (rightly) to set any limit for the beginnings of mind. We reached the same result in our consideration of the atom.

In summary, these lectures have tried to teach two lessons: "(1) that physics, physiology and psychology are inevitably bound up with one another, so that each one finds its salvation only in the companionship of the others, and each one may be regarded as handmaid of the others; and (2) that we are not bound to despair of exact investigation when we pass beyond the borders of the palpable."

Herbart and Wundt

Postscript.—Wundt has earned our gratitude by reducing the many riddles of the psychology of perception to a single riddle. But he is unfair to Herbart. (*a*) If the principle had always held good that science cannot be developed out of metaphysical hypotheses, we should never have had a mechanics, to say nothing of a psychology. (*b*) Those who know the Herbartian psychology assure us that it contains much more than could be discovered by mere attentive observation. (*c*) Wundt's criticisms in the *Beiträge* confuse intensity and clearness of ideas. Herbartians themselves do not always escape this error. (*d*) Wundt's experimental disproof of Herbart's two-idea minimum fails: it is as impossible to experiment with two ideas as with two atoms; and Wundt confuses idea with sensation. If Wundt's explanation of his complication-experiment were correct, we should have a mosaic of ideas, but nothing like connected thought.—Herbart has in fact made a very noteworthy contribution to scientific theory. It is no reproof to a young science to say that it proceeds on too simple assumptions. As a rule, Wundt's enquiries begin where Herbart leaves off.